

Extraction of Lignin from Hydrolyzed Lignocellulose

RAPHAEL KATZEN, FREDERICK G. SAWYER,
AND DONALD F. OTHMER

Polytechnic Institute of Brooklyn, N. Y.

The solubility characteristics of lignin present in acid-hydrolyzed lignocellulose were observed. Methanol was chosen as the most practical solvent for the separation and recovery of soluble lignin. Batch extraction experiments with methanol yielded lignin solutions of low concentration. The method of extraction, however, did not affect appreciably the nature of the lignin. Ninety-eight per cent of the lignin extracted was recovered by concentrating the extract to 25% lignin content and precipitating in 3 parts of water per part of concentrate. Ternary equilib-

EARLIER work (1) indicated that rapid hydrolysis of wood, carried out in a continuous process, yielded hydrolyzed lignocellulose containing a lignin fraction soluble in methanol. The amount of soluble lignin varied with the conditions of hydrolysis and showed a considerable influence on the properties of the hydrolyzed product, particularly when utilized in plastics. Initial work indicated that this fraction was practically unaltered chemically from the composition of previously isolated lignins; since the method of production has little effect on the degree of polymerization, it was deemed desirable to develop a practicable method for the separation and recovery of this soluble lignin.

There are few data in the literature on the solubility of lignin in various solvents, mainly because it is difficult to isolate a satisfactory material on which to base such data. There is also a lack of distribution data or methods of obtaining them for lignin between solvents and lignocellulosic materials. The present investigation was carried out to obtain fundamental information which would serve as a basis for the production of substantially carbohydrate-free lignin from hydrolyzed lignocellulose.

All data reported were based on partially hydrolyzed lignocellulose made from maple sawdust by the continuous process (1). The sample numbers refer to pilot plant hydrolysis runs and bear no relation to the composition of the materials. In general, the samples used for this work were hydrolyzed to contain 50-60% total lignin.

SOXHLET EXTRACTIONS

Samples of lignocellulose 328 containing 55.0% total lignin were extracted with various boiling solvents at atmospheric pressure in Soxhlet type apparatus. Complete extraction was shown by lack of color in the solvent after passing through the extraction thimble. The amount of lignin extracted was determined by drying the thimble and contents at 105° C. and finding the loss in weight, calculated on the basis of original lignocellulose and on the basis of total lignin content by the 72% sulfuric acid method (3). Data for the solvents tested are listed in Table I. Figure 1 is a bar graph of the data for the lower alcohols.

Among the alcohols, the lower members are the best lignin solvents. The high values for normal and isobutyl alcohols are offset by signs of degradation of the lignin; the possibility is also indicated that the lignin-cellulose linkage itself is attacked to

rium data were obtained for the system methanol-methanol-soluble lignin-insoluble lignocellulose. Application of continuous countercurrent extraction methods based on these data yielded extracts containing as high as 11% lignin. Higher concentrations were shown to be feasible by re-use of the extract on fresh lignocellulose feed. Calculations are presented, indicating a steam consumption of 2500 pounds for solvent recovery per 100 pounds of lignin recovered. Actual methanol losses could not be determined in the apparatus used.

free more lignin. Higher-boiling alcohols (2-ethyl hexyl and cyclohexyl) attacked the lignin and lignocellulose so severely that satisfactory data could not be obtained. After allowance for excessive values caused by degradation, it may be concluded that lignin solubility decreases with increase in the number of carbon atoms in monohydric alcohols. The lignin solubility also decreases with increasing number or complexity of the side chains in the alcohol molecule.

The simple esters are poor lignin solvents, but dioxane is comparable with methanol as a lignin solvent. The ethylene glycol monoalkyl ethers are even better; this is surprising, in view of the fact that no decomposition was noted with these solvents, despite their elevated boiling points. Apparently, then, lignin degradation, decomposition, or cleavage is not necessarily a function of temperature alone, but is affected by the nature and structure of the solvent. This information emphasizes the sensitivity of the lignin molecule or building unit to mild or non-reactive materials.

More reactive solvents such as diacetone alcohol also yield degraded lignins; an even more reactive solvent, ethylene chlor-

TABLE I. SOXHLET EXTRACTION OF LIGNIN

Solvent	Boiling Point, ° C.	% Lignin Dissolved, Based on:	
		Hydrolyzed wood	Total lignin in hydrolyzed wood
Methyl alcohol	65	20.00	36.35
Ethyl alcohol	78	19.65	35.70
<i>n</i> -Propyl alcohol	97	18.85	34.30
Isopropyl alcohol	83	15.36	27.90
<i>n</i> -Butyl alcohol	116	20.92 ^a	38.00 ^a
Isobutyl alcohol	108	16.90 ^a	30.70 ^a
<i>sec</i> -Butyl alcohol	99	11.10	20.20
<i>tert</i> -Butyl alcohol	83	6.48	11.80
2-Ethyl hexyl alcohol	186	a	a
Cyclohexyl alcohol	160	a	a
Methyl acetate	57	10.80	19.65
Ethyl acetate	77	8.78	15.96
Dioxane	102	21.30	38.70
Ethylene glycol mono-methyl ether	124	28.00	50.90
Ethylene glycol mono-ethyl ether	134	27.40	49.80
Ethylene glycol mono-butyl ether	169	28.20	51.25
Diacetone alcohol	184	17.90 ^a	32.50 ^a
Ethylene chlorhydrin	127	86.00 ^a	156.2 ^a

^a Degradation of lignin indicated by evolution of noncondensable gases or precipitation of resinous material from the extract.

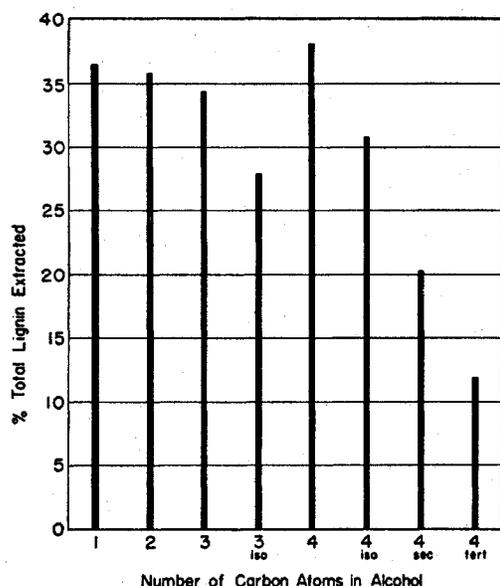


Figure 1. Extraction of Lignin with Boiling Alcohols (760 Mm.)

hydrin, evidently decomposes cellulose as well as lignin. Qualitative experiments indicated that acetone and ethyl ether were relatively poor solvents for lignin.

On the basis of good solvent properties, lack of reactivity, low boiling point, low cost, and ease of recovery, further work was carried on with methanol as the solvent for lignin.

SOLUBILITY OF LIGNIN IN METHANOL

In order to check the effect of temperature on the solubility of lignin in methanol, one-gram samples of lignocellulose 333 (methanol-soluble lignin content, 22.9%) were mixed thoroughly with 10 grams of methanol in a constant-temperature bath for 30 minutes at temperature up to 50° C. The extract was removed, centrifuged to clear it of suspended solids, weighed, and dried at 50° C. for 48 hours, and the lignin residue was weighed. Figure 2 gives the results.

Lignin solubility in methanol is limited at room temperature but rises rapidly at 35–40° C. The apparent leveling off at 50° C. was due only to limits of the amount of methanol-soluble lignin in the sample. It is indicated that at 50° C. and above, lignin solubility in methanol is not a limitation in the extraction process.

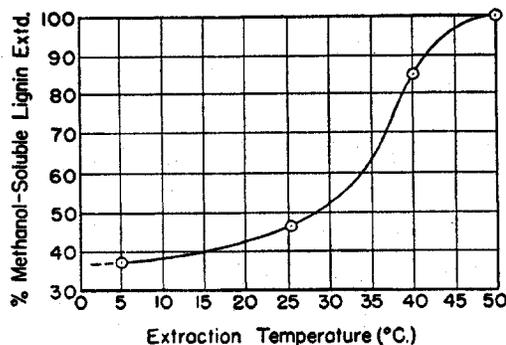


Figure 2. Effect of Temperature on Solubility of Lignin in Methanol

Temp., °C.	Gram Lignin/Gram Methanol	% of Methanol-Sol. Lignin Extracted
5.0	0.0086	37.7
25.5	0.0107	46.6
40.0	0.0195	85.1
50.0	0.0229	100.0

As further proof, a methanol-lignin solution concentrated to 28% lignin content by evaporation of solvent was used to extract lignin from lignocellulose 326 (methanol-soluble lignin content, 21.2%). At the end of 24 hours at room temperature, the lignin content had increased to 29.5% and after an additional 48 hours, to 30.0%; all due precautions had been taken to prevent evaporation of methanol.

PERCOLATION EXPERIMENTS

In an attempt to develop a batch method of extraction which would yield higher concentrations of lignin in the solvent, a Silex coffee percolator was charged with 1000 ml. of methanol in the lower chamber, and 300 grams of lignocellulose 326 wetted with 600 ml. of methanol (previously determined as the minimum retention) in the upper chamber.

Ten successive percolations with the same batch of methanol were then made, 10 ml. being withdrawn each time and tested for lignin content by drying and weighing. Presentation of the data in Figure 3 indicates the probability that the extracted lignin is a heterogeneous material, one part being extracted at a higher rate than the other. The semilogarithmic plot also shows that time, and not concentration, is the limiting factor in this extraction.

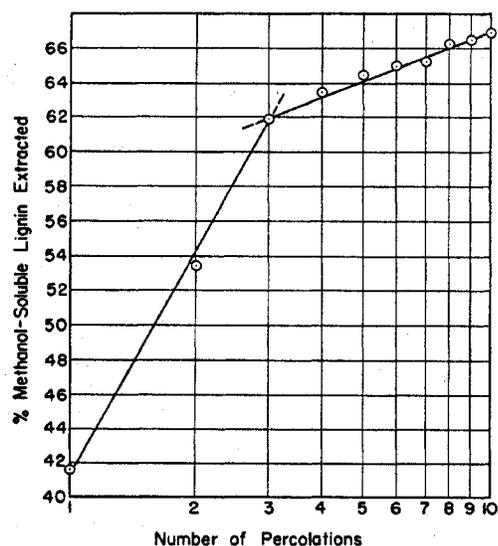


Figure 3. Rate of Extraction of Lignin with Methanol

No. of Percolations	Gram of Lignin/Ml. Ext.	% of Methanol-Sol. Lignin Extracted
1	0.0166	41.7
2	0.0214	53.5
3	0.0249	61.9
4	0.0258	63.5
5	0.0264	64.5
6	0.0267	65.0
7	0.0270	65.3
8	0.0276	66.3
9	0.0278	66.5
10	0.0282	66.9

Because of the relatively poor efficiency of extraction in this experiment, it was decided to utilize the most efficient part of the curve—namely, the first five percolations (allowing a safety factor)—and to use multiple batches of solvent.

Data of Figure 4 cover extraction of another batch of lignocellulose 326 under the conditions noted above, except that after each five percolations the extract in the lower chamber was withdrawn and a fresh batch of 1000 ml. of methanol was added. Lignin extraction was determined by concentrating each batch of extract to 25–35% lignin content by evaporation, pouring the concentrate into three times its own volume of slightly acidified

water, filtering, drying the residue at 50° C. for 48 hours, and weighing.

The straight-line semilogarithmic plot again indicates the time limitation on lignin extraction. With poor contact and agitation inherent in the percolation apparatus, ten batches of methanol would be required to remove 90% of the lignin. The resulting low lignin concentration would result in high recovery costs.

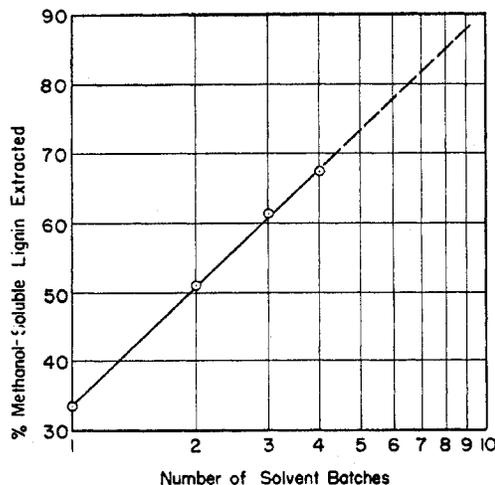


Figure 4. Multiple Batch Extraction of Lignin with Methanol

No. of Methanol Batches	Lignin Extd., G.	Cumulative Lignin Extd., G.	% Methanol-Sol. Lignin Extd.
1	21.0	21.0	33.1
2	11.5	32.5	51.1
3	6.4	38.9	61.2
4	3.7	42.6	67.0

A check on the methoxyl contents of the lignin fractions obtained from the four solvent batches yielded values of 19.5, 18.7, 19.2, and 19.7%, in that order. With known values of 20–21% methoxyl content for maple lignin, and the tendency for the acid hydrolysis treatment to split off some methoxyl from lignin, it may be concluded that the products obtained are essentially carbohydrate-free lignin and are closely related in composition. The differences in solubility, therefore, may be attributable to the varying degrees of polymerization of the lignin fractions.

A material balance was drawn up on this run. Analyses of the raw material and extracted residue were:

Ligno-cellulose	% Methanol-Sol. Lignin	% Insol. Lignin	% Cellulose (by Difference)
Original	21.2	33.4	45.4
Extracted	8.1	38.5	53.4

On a methanol-soluble lignin-free basis, the insoluble lignin is 42.3% for the original lignocellulose and 42.0% for the extracted lignocellulose. This proves that there was no appreciable effect of the method of extraction on the lignin-cellulose linkage or on the degree of polymerization of the insoluble lignin during ex-

traction. From the analyses, 14.5% of the original lignocellulose was removed by the methanol, or 43.5 grams. Lignin recovered by precipitation in water was 42.6 grams or 98%. This proves that there was little degradation to water-soluble fractions.

LIGNIN PRECIPITATION AND RECOVERY

Optimum conditions for precipitation and recovery of the lignin from the extract were determined. A dilute extract yielded finely divided, almost colloidal lignin upon mixing with cold water. On the other hand, highly concentrated extracts resulted in precipitation of amorphous, tarry lignin when mixed with water. An optimum concentration of 25–35% lignin in methanol yielded a fluffy, light-brown, easily filterable lignin.

The amount of water used in precipitation was controlled, as excessive amounts yielded dilute methanol filtrates which resulted in uneconomic methanol recovery. Insufficient water resulted in incomplete precipitation of lignin, as the more concentrated methanol-water mixture retained some solvent power for lignin. A volume ratio of 1 part extract to 3 parts water was found to be a good compromise; lignin precipitation was complete, and a fairly concentrated methanol solution was obtained for recovery.

DISTRIBUTION DATA

Since initial batch extraction methods had proved inefficient as a result of excessive dilution of the extract when appreciable proportions of the methanol-soluble lignin were dissolved, development of a continuous countercurrent method was undertaken. To obtain fundamental data for design calculations, a ternary distribution plot was set up on the basis of the three-phase system (two solid, one liquid) soluble lignin-insoluble lignocellulose-methanol.

Samples of lignocellulose 333 (22.9% methanol-soluble lignin) were mixed with varying amounts of methanol, the mixture was held at 50° C. for 30 minutes, the extract was decanted and the methanol and lignin contents were determined by evaporating to dryness at 50° C. for 48 hours. It was found that the maximum retention of methanol by lignocellulose with this procedure was 3 parts methanol to 1 part of solid. From this point ratios of solvent to solid were increased up to 40:1. Table II gives data for the material balances in these determinations; actual distribution data are listed in Table III and plotted in Figure 5.

In the ternary diagram, feed compositions for methanol-lignocellulose 333 mixtures are indicated by the line starting at the 100% methanol apex and extending toward the point 22.9% methanol-soluble lignin, 77.1% insoluble lignocellulose on the left-hand side. Extract compositions for any given mixture are located along the methanol-soluble lignin line on the right. Corresponding residue compositions determine the shape of the curve under the feed line, which may be designated a "retention" curve. This curve reaches a minimum methanol value, indicating that the amount of soluble lignin retained has some effect on the methanol retention by the insoluble lignocellulose.

The intersection of the retention curve and the feed line represents the 3:1 methanol:lignocellulose mixture. Extension of a tie line parallel to the known tie lines from this point to the extract line indicates a retained extract concentration of 7.8%. By calculation based on the lignocellulose analysis, this composition should be 7.6%. Similar checking of the other extract concentrations showed that extraction of methanol-

TABLE II. EFFECT OF METHANOL-LIGNOCELLULOSE RATIO ON EXTRACTION OF LIGNIN

Ratio, Methanol:Ligno-cellulose	Ratio, Extract:Ligno-cellulose	Concn. of Lignin in Ext., %	Separated in Ext.		Ratio, Lignin Separated: Methanol Separated	Concn. of Methanol in Residue, %		Ratio, Extract: Residue	
			% methanol-sol. lignin	% of methanol		Exptl.	Calcd.	Exptl.	Calcd.
3.5:1	1.73:1	6.93	52.3	46.0	1.14	68.3	68.2	0.62:1	0.59:1
4:1	2.41:1	6.28	66.2	56.5	1.17	67.2	67.1	0.93:1	0.85:1
5:1	3.54:1	4.95	76.5	68.4	1.12	66.4	65.7	1.44:1	1.45:1
8:1	6.16:1	3.21	86.5	74.6	1.12	71.4	71.7	2.17:1	2.33:1
10:1	7.99:1	2.47	86.3	77.9	1.18	73.3	73.4	2.66:1	2.84:1
20:1	17.4:1	1.29	98.0	85.8	1.14	78.3	78.6	4.84:1	4.79:1
40:1	34.4:1	0.77	116	85.3	1.36	88.2	89.1	5.25:1	5.21:1

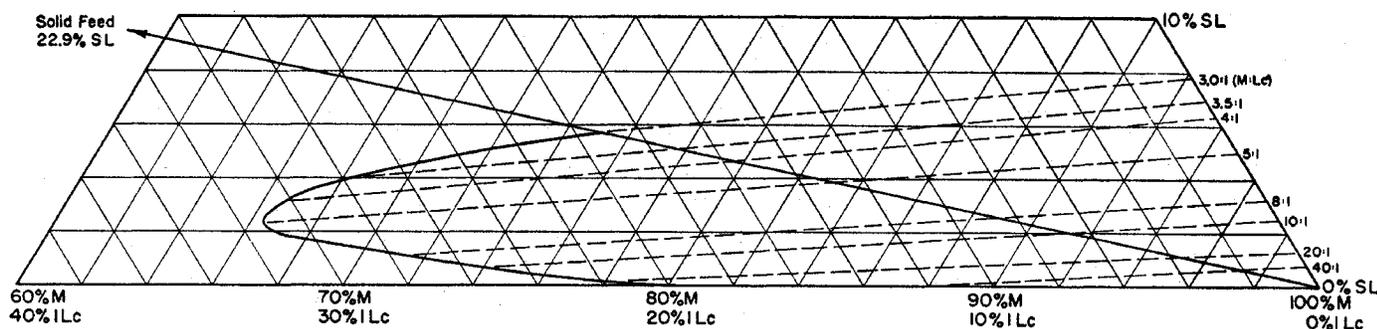


Figure 5. Distribution Equilibria for the System Methanol-Methanol-Soluble Lignin-Insoluble Lignocellulose

TABLE III. TERNARY EXTRACTION DATA FOR SYSTEM METHANOL (M)-SOLUBLE LIGNIN (SL)-INSOLUBLE LIGNOCELLULOSE (ILc)

Ratio, Methanol: Ligno-cellulose	Compn. of Mixture, %			Compn. of Extract, %		Compn. of Residue, %		
	M	SL	ILc	M	SL	M	SL	ILc
3.5:1	77.8	5.1	17.1	93.1	6.9	68.3	4.0	27.7
4:1	80.0	4.6	15.4	93.7	6.3	67.2	3.1	29.7
5:1	83.4	3.8	12.8	95.0	5.0	66.4	2.3	31.3
8:1	88.9	2.5	8.6	96.8	3.2	71.4	1.1	27.5
10:1	90.9	2.1	7.0	97.5	2.5	73.3	1.0	25.7
20:1	95.2	1.1	3.7	98.7	1.3	78.3	0.1	21.6
40:1	97.5	0.6	1.9	99.2	0.8	88.2	0	11.8

soluble lignin was complete in every instance. Discrepancies between experimental and calculated values can be accounted for as errors in plotting the experimental points and the curve.

These data signify that extraction of soluble lignin from lignocellulose with methanol is an unidirectional process, not subject to any equilibrium limits. This would indicate further that the methanol-soluble fraction is not chemically but is physically combined with the balance of the lignocellulose.

Although all of the soluble lignin dissolved in the methanol, not all of it is readily recoverable. A large proportion of the dissolved lignin is retained along with the methanol in the residue. Much of the retained extract can be recovered by pressing the residue, but in a countercurrent system it would be removed by contact with more dilute extract. The extract concentration is not limited to 7.8%, as lignocellulose could be mixed with a higher concentration of extract and solution of lignin would still occur. In this case the feed line would run from the initial extract composition point on the methanol-soluble lignin line to the same lignocellulose composition point noted on the left-hand side of Figure 5.

CONTINUOUS EXTRACTION

An experimental unit was set up for continuous extraction of lignin with methanol. A vertical Pyrex tube, 40 inches long and $\frac{3}{4}$ inch in inside diameter, was equipped with a spiral-screwfeed device at the top to feed 20-mesh lignocellulose 333. The extract overflowed just below the solid feed point, but solids carry-over was minimized by use of an inner concentric glass tube immersed to a depth of 6 inches in the methanol. Methanol was provided by evaporating solvent continuously from an electrically heated flask, condensing, and returning the hot liquid through a trap to the bottom of the extractor. Agitation was maintained in early runs by a stream of compressed air entering the extractor through a perforated glass distributor placed above the methanol feed point. Excessive cooling of the methanol, loss of solvent, and carry-over of lignocellulose particles by the air stream led to modification of the operation, in which methanol vapor and liquid were fed to the extractor. The vapor served to agitate the solids in the column, maintained the methanol at the boiling point at the base of the column, and was finally condensed and

cooled by an air stream carried across the top of the extractor. Data for the runs listed in Table IV were obtained only after the extractor had been brought into balance for a given set of conditions. Extracts were measured and tested for lignin content over the period required to feed 20-gram lignocellulose samples, the feed being maintained as constant as possible with a manually controlled feed device.

Examination of Table IV shows that agitation is most important for extraction of lignin from lignocellulose. Use of even a moderate flow of air (25 ml. per minute) increased substantially the proportion of lignin extracted. The feed rate of lignocellulose did decrease, but the reduction was not sufficient to account for increased extraction as a function of retention time of lignocellulose in the column. Higher rates of air flow indicated a peak value which would effect maximum extraction of the lignin under the operating conditions at about 325 ml. per minute. Higher flow rates probably decrease extraction efficiency by lowering excessively the methanol temperature.

TABLE IV. CONTINUOUS EXTRACTION RUNS

Feed Rate per Min.	Extract Rate, G./Min.	Extract Concn., % Lignin	% of Lignin Extd.	
Air, ml.	Lignocellulose, grams			
0	1.05	24.9	0.27	27.6
25	0.86	25.3	0.40	51.5
325	1.14	25.5	0.63	61.0
400	1.25	25.0	0.60	53.2
Methanol Vapor Agitation				
...	2.11	35.6	0.68	48.9
...	1.51	27.7	0.87	68.5

The first run made with methanol vapor agitation was evidently fed at too fast a rate. The second run, however, although faster in lignocellulose feed rate than the foregoing air-agitated runs, was considerably more efficient in lignin extraction; 68.5% of the soluble lignin in the lignocellulose was removed. This value is comparable with results of the ten percolation and four multiple-batch percolation extractions. Lignin concentration in the extract, although higher in the last continuous extraction run than in the preceding ones, is still too low for practical purposes. This factor was limited by lignocellulose and methanol feed rates. The former could not be increased because the capacity of the extractor was small; the latter could not be decreased appreciably without interrupting continuous flow of solvent to the extractor with the feed system.

Similar operation in a larger continuous extractor (1.75 inches i.d. and 16 feet 2 inches high) yielded extracts containing as high as 11% lignin.

STEAM CONSUMPTION FOR METHANOL RECOVERY

As an indication of the economic feasibility of the process developed, calculations were made for steam requirements in methanol recovery. Consumption of steam per 100 pounds of lignin

recovered was based on the following assumptions: 10% concentration of lignin in extract; 90% of methanol-soluble lignin dissolved in methanol extract, 10% retained by lignocellulose; use of lignocellulose 256, containing 33.3% methanol-soluble lignin; methanol recovered at 98.2 mole % concentration; spent water containing less than 0.01% methanol; and steam supply at 5 pounds per square inch gage.

Lignin extract is concentrated to 25% lignin content by evaporation of methanol. The concentrate is poured into three times its own weight of water, yielding a 25% methanol filtrate (15.8 mole %). With feed preheated by spent water to 156° F. and referring to the McCabe-Thiele method (2):

$$\begin{aligned} q &= 1.043 \\ \text{slope of } q\text{-line} &= 24.3 \\ \text{minimum reflux} &= 1.04 \\ \text{practical minimum reflux} &= 1.04 \times 1.5 = 1.56 \\ V/D &= 2.56 \end{aligned}$$

Approximately twenty theoretical plates are required, according to the methanol-water vapor liquid equilibrium diagram. The heat requirements are:

Evaporation of methanol	313,170 B.t.u.
Distillation of filtrate	415,400
Distillation of washings ¹	1,661,600
Total	2,390,170 B.t.u.

$$\frac{2,390,170}{960} = 2500 \text{ lb. steam per 100 lb. lignin recovered}$$

¹ Methanol retained in lignocellulose recovered by countercurrent washing with water, yielding a 25% methanol solution.

CONCLUSIONS

Although the ethylene glycol monoalkyl ethers were the best solvents tested as extractants of lignin from acid-hydrolyzed lignocellulose, methanol was chosen as the most practical because of its low cost, ease of recovery, and good solvent power.

Extraction of lignin with methanol is facilitated with increased temperatures, and the process is a function of time of contact alone.

A method was developed for determination of distribution data in the ternary system methanol-soluble lignin-insoluble lignocellulose. Although individual diagrams must be drawn up for hydrolyzed lignocelluloses according to their soluble lignin content, the one diagram illustrated indicates the general properties of this liquid-solid-solid system. Another variation in the system is possible through use of recirculated extracts at higher soluble-lignin contents; the necessity is thereby eliminated of evaporating any of the methanol before lignin precipitation. Such work was beyond the scope of the present research.

Optimum conditions were set up for precipitation and recovery of the lignin extracted, and data were presented indicating that, although the lignin is practically unchanged in composition (particularly methoxyl content) by the method of extraction, the products obtained are probably part of a polymeric series.

Operation of a continuous laboratory extractor has shown the feasibility of continuous operation, although equipment must be developed to feed lignocellulose at a uniform rate. A larger extractor yielded extracts containing as high as 11% lignin.

Calculations on methanol recovery show a steam consumption of 2500 pounds per 100 pounds of lignin recovered, but pilot plant operations will be required to determine methanol losses, which are the most critical factors in the economy of the process. (The process described is the subject of patent applications.)

ACKNOWLEDGMENT

The authors express their appreciation and thanks to the Northwood Chemical Company for establishing the fellowship on which this work was accomplished.

LITERATURE CITED

- (1) Katzen, R., and Othmer, D. F., *IND. ENG. CHEM.*, 34, 314 (1942).
- (2) McCabe, W. L., and Thiele, E. W., *Ibid.*, 17, 605 (1925).
- (3) Ritter, G. J., Seborg, R. M., and Mitchell, R. L., *IND. ENG. CHEM., ANAL. ED.*, 4, 202 (1932).

Nomograph for Absorption Factor Equation

GEO. E. MAPSTONE

National Oil Proprietary Ltd.,
Glen Davis, N.S.W., Australia

CERTAIN methods for the design of countercurrent absorbers (1, 2) involve the use of the absorption factor equation

$$E = \frac{A^{N+1} - A}{A^{N+1} - 1}$$

where E = "efficiency" of operation
 A = absorption factor
 N = number of theoretical plates in system

Figure 1 is developed from the transformed equation,

$$1 - E = \frac{A - 1}{A^{N+1} - 1}$$

by the combination of the two simplest types of nomograph—namely, the Z -type and the logarithmic type.

Because of the kind of equation involved, no reading is obtainable from the chart when $A = 1$. In this case, however,

$$E = \frac{N}{N+1} \text{ or } N = \frac{E}{1-E}$$

The nomograph can be used to solve for any of the variables E , N , or A , the other two being known:

METHOD I. To solve for E , A and N being given: Connect the values of A and N to give A^{N+1} . Make the simple calculations $(A - 1)$ and $(A^{N+1} - 1)$. Connect the values of $(A - 1)$ and $(A^{N+1} - 1)$ to intersect the E scale at the appropriate value of E .

METHOD II. To solve for N , A and E being given: Make the simple calculation $(A - 1)$. Connect $(A - 1)$ and E , and extend to give the appropriate value of $(A^{N+1} - 1)$. Compute A^{N+1} from the value of $(A^{N+1} - 1)$. Connect A and A^{N+1} to obtain the appropriate value of N .

METHOD III. To solve for A , N and E being given: In this case the solution is by trial and error. Successive values of A are chosen, and method I is applied until a value of A is found which, when the known value of N is used, gives the known value of E .

Occasionally values of A or $A - 1$ are required which do not appear on the scale of the chart. In order to handle these values,